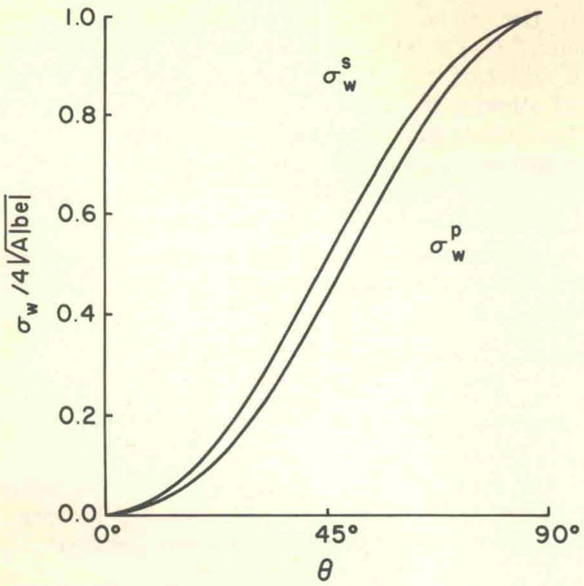


FIG. 3. Domain-wall energy as a function of $\theta$. $\sigma_w^s$ corresponds to walls shown in Fig. 2(a). $\sigma_w^p$ corresponds to walls shown in Fig. 2(b).

is usually sufficient. The $\alpha_K$'s are the direction cosines of the magnetization vector. That is, $M_K = M_s\alpha_K$, where $M_s$ is the saturation magnetization of the ferromagnetic material. For cubic symmetry, this expression reduces to

$$E_{ex} = A[(\nabla\alpha_1)^2 + (\nabla\alpha_2)^2 + (\nabla\alpha_3)^2], \quad (4)$$

where $A$ is the exchange constant.

The fourth term is the crystalline anisotropy energy. From conventional magnetoelastic theory, it is given by

$$E_K = K_{ijkl}\alpha_i\alpha_j\alpha_k\alpha_l.$$

For cubic symmetry, it becomes

$$E_K = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2). \quad (5)$$

In this paper, interest lies in the shock-induced anisotropy. In shock-wave studies, strains in the large elastic and plastic regions are obtained.[12] For many magnetic materials, the crystalline anisotropy energy is 10–30 times smaller than the induced anisotropy energy in this strain region. For this reason the crystalline anisotropy will be ignored.

The last term is the magnetoelastic energy. From conventional magnetoelastic theory, it is given by

$$E_{me} = b_{ijkl}e_{ij}\alpha_k\alpha_l, \quad (6)$$

where $b_{ijkl}$ is the fourth-rank magnetoelastic tensor. For cubic symmetry, this becomes

$$E_{me} = b_1(\alpha_1^2 e_{11} + \alpha_2^2 e_{22} + \alpha_3^2 e_{33}) + 2b_2(\alpha_1\alpha_2 e_{12} + \alpha_2\alpha_3 e_{23} + \alpha_3\alpha_1 e_{31}). \quad (7)$$

The magnetoelastic energy is of primary interest in the shock-induced anisotropy effect.

For a single-crystal slab of ferromagnetic material with crystal axis arbitrarily oriented with respect to the axis of uniaxial strain, the strain tensor can be written

$$e_{ij} = en_in_j, \quad (8)$$

where $n_j$ is a component of a unit vector directed along the axis of uniaxial strain. $e = (\rho_0/\rho) - 1$ is the strain along this axis, where $\rho_0$ and $\rho$ are the initial and final densities, respectively. The magnetoelastic energy [Eq. (6)] becomes

$$E_{me} = eb_{ijkl}n_in_j\alpha_k\alpha_l. \quad (9)$$

This can be written

$$E_{me} = C_{kl}\alpha_k\alpha_l, \quad (10)$$

where

$$C_{kl} = eb_{ijkl}n_in_j. \quad (11)$$

This manipulation is very convenient since it allows the familiar techniques developed for analyzing symmetric second-rank tensors to be used in analyzing the fourth-rank magnetoelastic tensor for a given state of uniaxial strain. For cubic symmetry the matrix array representing the second rank tensor in Eq. (11) becomes

$$[C_{kl}] = e\begin{pmatrix} b_1n_1^2 & b_2n_1n_2 & b_2n_1n_3 \\ b_2n_1n_2 & b_1n_2^2 & b_2n_2n_3 \\ b_2n_1n_2 & b_2n_2n_3 & b_1n_3^2 \end{pmatrix}. \quad (12)$$

The principal axes of the representation quadric for this second-rank symmetric tensor give the easy and hard directions of magnetization produced by the induced uniaxial strain. The eigenvalues are the magnetoelastic energies when the magnetization vector lies along the corresponding principal axes.[17] It should be noted that the principal axes depend only on the direction of the axis of uniaxial strain with respect to the crystal axes and not on the magnitude of strain since the eigenvectors will be functions of the $n_i$ and independent of $e$. Of more interest is the fact that the axis of uniaxial strain will not, in general, coincide with a principal axis and hence will not define an easy or hard direction of magnetization. There are special cases, such as uniaxial strain along a ⟨100⟩ or a ⟨111⟩ axis, in which the strain axis and a principal axis coincide. This has the following implication: First-order conventional magnetoelastic theory[16] predicts that in any finite magnetic field, strain-induced anisotropy cannot produce

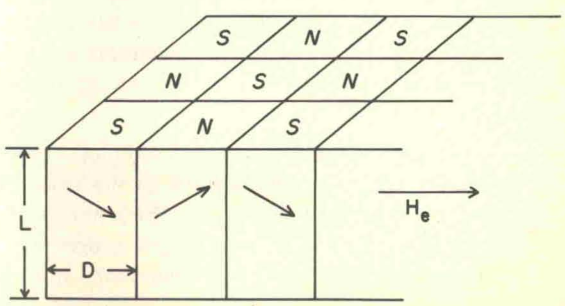


FIG. 4. Surface pole distribution for magnetostatic potential problem. There is pole distribution on both upper and lower surfaces.

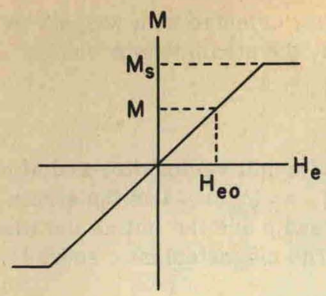


FIG. 5. Predicted magnetization curve. The magnetization $M$ determines the extent of the shock-induced demagnetization that would occur for a specimen initially saturated in a field $H_{e0}$.

total shock demagnetization except in the special cases stated previously. This can be seen in the following way: The geometry of the shock demagnetization problem (Fig. 1) defines an axis of uniaxial strain with a perpendicular applied field. The single-crystal axes may be arbitrarily oriented. The direction of easy magnetization will not, in general, coincide with the axis of uniaxial strain. In the limit of vanishingly small applied field $\mathbf{H}_e$, the magnetization will lie along this easy axis. Its direction along this axis will be such that $-\mathbf{H}_e \cdot \mathbf{M}_s$ is minimal. This will give a nonzero component of $\mathbf{M}$ in the direction of $\mathbf{H}_e$. In a polycrystalline material all orientations of crystallites occur. Each will contribute to the transverse magnetization. This may explain, at least in part, why shock-induced demagnetization observed by Shaner and Royce[5] in YIG was less than expected.

## III. DOMAIN-THEORY ANALYSIS

In the domain-theory analysis of shock-induced anisotropy, two single-crystal problems will be treated concurrently. These will be called the ⟨100⟩ problem and the ⟨111⟩ problem. The ⟨100⟩ problem corresponds to a state of uniaxial strain along a ⟨100⟩ axis with a perpendicular applied field. The ⟨111⟩ problem corresponds to a state of uniaxial strain along a ⟨111⟩ axis with a perpendicular applied field. These two problems have been chosen for the following reasons: In single-crystal magnetostriction, inverse of the effect considered in the present work, results are interpreted in terms of $\lambda_{100}$ and $\lambda_{111}$. These magnetostriction constants represent total strain when a crystal is magnetized from the demagnetized state to saturation along the ⟨100⟩ and ⟨111⟩ axes. The problems considered in the present work are the complementary analogs of these inverse magnetostriction problems. The results clearly exhibit characteristic behavior of the shock-induced anisotropy effect. Also these results will be used in determining polycrystalline magnetic behavior.

There is an inherent weakness in ferromagnetic domain theory. A basic postulate of the theory is the existence of domain walls. However, the theory does not provide a means for determining unambiguously the domain structure for a given problem. The procedure is to assume possible domain structures consistent with other requirements of the problem and select from these, by energy considerations, the most likely domain structure. In Fig. 2, models for domain structures consistent with requirements of the present problem are shown. Domain walls normal to the strain axis are not expected. This is because the variation in the magnetization direction through the domain wall cannot be made without allowing $\nabla \cdot \mathbf{M}$ to deviate from zero. $\nabla \cdot \mathbf{M} \neq 0$ in the domain wall implies magnetic volume poles in the wall which would contribute excessively to the demagnetizing energy. This would be energetically unfavorable. That $\nabla \cdot \mathbf{M} = 0$ through the domain wall is a postulate of ferromagnetic domain theory. Also, domains of closure are not expected due to the very large induced anisotropy energy.

### A. Induced Anisotropy Energy

The induced anisotropy energies for the ⟨100⟩ problem and the ⟨111⟩ problem will be obtained in this section. The energy will be obtained for the region within domains and within the walls through which the transition between adjacent domains is made. This will be done for walls of the form shown in Figs. 2(a) and 2(b).

Consider first the ⟨100⟩ problem and the domain geometry in Fig. 2(a). Transform Eq. (7) to polar coordinates using Eq. (8),

$$\alpha_1 = \sin\theta\cos\Phi, \quad \alpha_2 = \sin\theta\sin\Phi, \quad \text{and } \alpha_3 = \cos\theta. \tag{13}$$

The induced anisotropy energy in a domain is easily obtained:

$$E_{\text{me}}^{\langle 100\rangle}(\text{domain}) = b_1 e \sin^2\theta. \tag{14}$$

To obtain the induced anisotropy energy in the wall the variation in $\mathbf{M}$ through the wall must be considered. The requirement that $\nabla \cdot \mathbf{M} = 0$ through the wall is equivalent to demanding that $\theta$ be constant through the wall. This requires the transition between adjacent domains to proceed by a rotation of $\Phi$ from 0 to $\pi$. The energy in the wall is

$$E_{\text{me}}^{\langle 100\rangle}(\text{wall}) = b_1 e \sin^2\theta\cos^2\Phi. \tag{15}$$

In determining Eqs. (14) and (15) from the geometry in Fig. 2(a), it should be pointed out that within a domain $\mathbf{M}$ lies in the $xz$ plane and, therefore, $M_y = 0$ or $\Phi = 0$; while in the wall, $\nabla \cdot \mathbf{M} = 0$ implies $\partial M_z/\partial z = 0$ and, therefore, $M_y \neq 0$ which implies $\Phi \neq 0$, i.e., $\mathbf{M}$ rotates out of the $xz$ plane in keeping $\nabla \cdot \mathbf{M} = 0$.

A slightly more difficult analysis gives for the ⟨111⟩ problem

$$E_{\text{me}}^{\langle 111\rangle}(\text{domain}) = b_2 e \sin^2\theta \tag{16}$$

and

$$E_{\text{me}}^{\langle 111\rangle}(\text{wall}) = b_2 e \sin^2\theta\cos^2\Phi. \tag{17}$$

This is most easily accomplished by subjecting the energy expression in Eq. (7) to a coordinate transformation such that the new $x$ axis lies along the old ⟨111⟩ direction. Since the forms of the energies are the same for the ⟨100⟩ problem and the ⟨111⟩ problem we will write

$$E_{\text{me}}(\text{domain}) = b e \sin^2\theta \tag{18}$$

and

$$E_{\text{me}}(\text{wall}) = b e \sin^2\theta\cos^2\Phi, \tag{19}$$